STAR TIME SERIES MODELS WITH ASYMMETRIC TRANSITION FUNCTION: ESTIMATION USING R SOFTWARE AND APPLICATIONS FOR LATVIAN ECONOMIC DATA

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Abstract. Smooth Transition Autoregressive (STAR) models are particularly effective for analysing data containing structural breaks. These models utilise various transition functions and threshold variables to accommodate distinct patterns of structural changes in time series. This study introduces asymmetry into STAR models by proposing a new asymmetric smooth transition function. All transition functions developed so far assume symmetry with respect to a threshold (in ESTAR and LSTAR) or a midpoint between two thresholds (in LSTAR2). However, in practical scenarios, transitions between regimes may occur at different rates, with regime change in one direction potentially being faster or slower than in the opposite. The proposed transition function includes two distinct, positive smoothness parameters. When the first parameter is larger (smaller) than the second, the regime transition happens more (less) rapidly, while the return transition is slower (faster). To practically implement the asymmetric smooth transition autoregressive (ASTAR) model and compare its performance with other STAR models, the estimation procedure of LSTAR in the R programming environment is modified for the ASTAR model. For the evaluation of the goodness of fit of the developed model on the real-life economic data, series of employment growth in Latvia are used. The findings suggest that the ASTAR model achieves a superior fit for employment growth data compared to traditional STAR models.

Keywords: time series, smooth transition autoregressive model, transition function, LSTAR, ESTAR.

Introduction

Time series analysis plays a crucial role in understanding and forecasting economic phenomena [1; 2]. Among the various models used for this purpose, Smooth Transition Autoregressive (STAR) models are considered as an effective tool for incorporating structural breaks and other nonlinearities by gradual regime switching. These models are based on transition functions that allow for smooth changes between different regimes, making them particularly useful for modelling data with structural shifts, especially in economic and financial data.

Initially, the **STAR model** was proposed by **Teräsvirta** in 1994 [3]. It was developed as an alternative to self-exciting threshold autoregressive (SETAR) models introduced by Tong in 1983 [4] which describe instantaneous switching between regimes. A **transition function** is a key component of a STAR model that controls how the model moves between different regimes. The transition function is typically a smooth function of some transition variable. At the beginning, the logistic and exponential transition functions were used giving the model names LSTAR and ESTAR respectively [3]. Over the years, some more forms of the STAR model were proposed, each aiming to improve the flexibility and accuracy of regime-switching models. The most popular of them is the second-order LSTAR, or LSTAR2 [5]. A useful comparison of transition functions can be found in [6]. Besides, an absolute logistic smooth transition function was offered by Liews [7]. A lot of papers in this area have been devoted to estimation, specification, testing, comparison, forecasting, and applications of these STAR models [6; 8-10].

The mentioned traditional STAR models are based on the assumption that the transition between regimes has some type of symmetry. However, in many real-world scenarios, the transition between regimes may occur at different rates in different directions, which these models do not account for. To remove this limitation, the present study introduces a new model – the **Asymmetric Smooth Transition Autoregressive (ASTAR)** model. The ASTAR model incorporates asymmetry into the transition process, allowing for faster or slower regime switching depending on the direction of change. The concept of the model is not completely new, it is in compliance with prior fundamental works such as Terasvirta [3], providing foundational insights into the specification and estimation of STAR models. The ASTAR model extends the previous models by incorporating two distinct smoothness parameters, which allow for a more flexible modelling of asymmetric transitions in economic data.

To implement the ASTAR model, we modify the estimation procedure of the LSTAR model in the R programming environment given in *tsDyn* package. Another modification of this procedure, for

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LSTAR2, is described in [10]. These modifications allow us to compare the performance of ASTAR with other STAR models in practical applications.

The model is tested on real Latvian economic data, namely employment growth, to evaluate its goodness of fit. The results of this study suggest that the ASTAR model provides a better fit for employment growth when compared to traditional STAR models. This finding demonstrates the potential of asymmetric transition models for more accurate economic forecasting and economic analysis.

This paper contributes to the growing research on nonlinear time series models by providing a novel model for asymmetric regime shifts, with particular application to Latvian economic data.

The rest of the paper is organised as follows. The section Materials and Methods introduces the STAR model, popular transition functions, and the new offered asymmetric function with a short description of its properties as well as the estimation procedure and the tests which help to choose the structure of the transition function. Also, this section describes the data used for the illustration of the usability of the models. The next section, Results and Discussion, presents the empirical results of the study. The section Conclusion contains the outcomes of the study.

Materials and methods

In general, a STAR model describes gradual transitions between linear autoregressive models, characterizing different regimes, regulated by some nonlinear transition function with values between 0 and 1. Its common structure may be given by formula (1):

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \dots + \alpha_p Y_{t-p} + \theta(\gamma, x, c)\left[\beta_0 + \beta_1 Y_{t-1} + \dots + \beta_p Y_{t-p}\right] + \varepsilon_t \quad (1)$$

where Y_t – variable of interest (univariate time series);
$Y_{t-1}, Y_{t-2}, \dots$ – lags of this variable;
α_i and $\beta_i, (i = 1, \dots, p)$ – autoregressive parameters;
ε_t – random error component with an assumption to be $\varepsilon_t \sim nid(0, \sigma^2)$;
$\theta(\gamma, x, c)$ – nonlinear transition function having values from the interval [0; 1];
γ – smoothness parameter (can be two parameters); x is a threshold variable;
$\boldsymbol{c}$ – threshold (can be two thresholds).

So, the model describes a gradual switching between the regimes shown by formulae (2)-(3):

Regime 1:

$$Y_t = \alpha_0 + \alpha_1 Y_{t-1} + \dots + \alpha_p Y_{t-p} \quad (2)$$

Regime 2:

$$Y_t = (\alpha_0 + \beta_0) + (\alpha_1 + \beta_1) Y_{t-1} + \dots + (\alpha_p + \beta_p) Y_{t-p} \quad (3)$$

As the smoothness parameter γ increases, the transition from one autoregressive equation to another and back becomes faster. The threshold variable can depend on one or more lags of the dependent variable (Y_{t-1},…), its differences (ΔY_{t-1}), the time variable or some other exogenous variable, possibly with lags.

The Logistic Smooth Transition AutoRegressive (LSTAR) model has the transition function (4):

$$\theta(\gamma, x, c) = \left[1 + e^{-\gamma(x-c)}\right]^{-1}. \quad (4)$$

The Exponential Smooth Transition AutoRegressive (ESTAR) model has the transition function (5):

$$\theta(\gamma, x, c) = 1 - e^{-\gamma(x-c)^2}, \gamma > 0 \quad (5)$$

The second order Logistic Smooth Transition AutoRegressive (LSTAR2) model has the transition function (6):

$$\theta(\gamma, x, c_1, c_2) = \frac{1}{1 + e^{-\gamma(x-c_1)(x-c_2)}}. \quad (6)$$

The typical shapes of these functions are shown in Fig.1

With increasing of the smoothness parameter γ, LSTAR tends to the SETAR model, ESTAR tends to the linear regression model, and LSTAR2 also tends to SETAR. The LSTAR transition function changes its values from 0 to 1 (and vice versa) passing the threshold. The ESTAR transition function temporarily falls from 1 to 0 passing the threshold and returns. The LSTAR2 transition function never distinguishes 0. Its minimum, when the threshold variable passes the threshold, is between 0 and 0.5. So, the regime of LSTAR2 in the threshold is

$$Y_t = (\alpha_0 + \min_x \theta \cdot \beta_0) + (\alpha_1 + \min_x \theta \cdot \beta_1)\, Y_{t-1} + \ldots + (\alpha_p + \min_x \theta \cdot \beta_p)\, Y_{t-p}$$

while

$$Y_t = (\alpha_0 + \beta_0) + (\alpha_1 + \beta_1)\, Y_{t-1} + \ldots + (\alpha_p + \beta_p)\, Y_{t-p}$$

in the opposite regime.

The offered asymmetric smooth transition function (ASTAR) is given by formula (7).

$$(c, x, \gamma_1, \gamma_2) = 1 - \frac{1.5}{1 + 0.5e^{-\gamma_1(x-c)} + 0.5e^{\gamma_2(x-c)}} \qquad (7)$$

It is dependent on the threshold variable x (for example, $x = Y_{t-1}$), some threshold value c, which is usually an unknown parameter that should be estimated, and two smoothness parameters γ_1, γ_2 which also should be estimated. Fig.2 shows the behaviour of the ASTAR transition function for different pairs of γ_1, γ_2 values and c = 0. As we can see from the graph, if γ_1 is large, but γ_2 is small, the first transition is faster, the second is much slower (in the case of the continuous purple line). The dashed purple line shows the opposite case.

The parameters γ_1, γ_2 should be of the same sign for the function value to be between 0 and 1, which is obligatory for a transition function. If both γ_1 and γ_2 are negative, then γ_2 refers to the first transition and γ_1 to the second. The ASTAR transition function may not reach 0 fully under some smoothness parameters similar to LSTAR2. Its minimum is between 0 and 0.25. The imperfection of this function is a small shift of the minimum from $x = c$ when $\gamma_1 \neq \gamma_2$, which is not significant while the difference between γ_1 and γ_2 is not unreasonably huge.

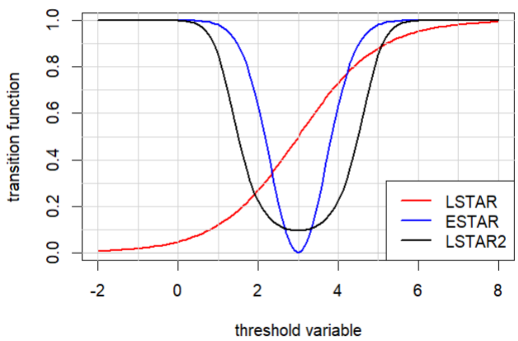

Fig. 1. **Transition functions of STAR models: LSTAR(c = 3,γ = 1), ESTAR(c = 3,γ = 1), LSTAR2 (c_1 = 1.5, c_2 = 4.5, γ = 1)**

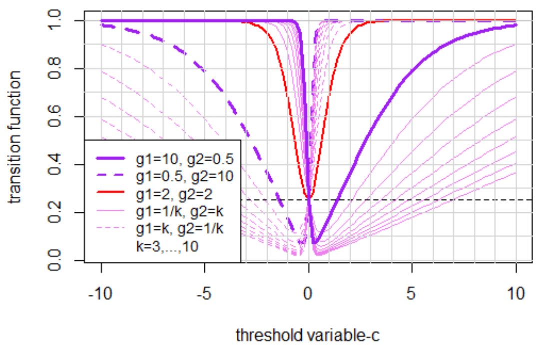

Fig. 2. **Transition function of ASTAR models with different smoothness parameters ($\gamma_1 = g_1$, $\gamma_2 = g_2$)**

The STAR models are typically estimated using conditional least squares. For a given set of parameters (thresholds and smoothness) of a transition function, the regime-specific coefficients can be estimated using Least Squares. The parameters of the transition function are identified through the grid search, minimizing the residual variance. It is 2-dimensional for LSTAR and ESTAR, and 3-dimensional for LSTAR2 and ASTAR. Nonlinear optimization is conducted using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) algorithm. So, all the parameters of the chosen model can be estimated using the given time series. The estimation and its difficulties are described by Terasvirta [3]. Terasvirta underlined [3] that even if the convergence is reached by estimating the parameters, then also the model's validity must be assessed. Because of the existence of local minima, especially if the time series is relatively short, it should be considered whether the estimates look reasonable. Autocorrelation of residuals should be inspected to choose the full and the most parsimonious model.

For the choice between STAR models, Terasvirta proposed a test [3], initially for the choice between LSTAR and ESTAR, based on the expansion of a transition function into the Taylor series by powers of a threshold variable and testing the coefficients near different powers using usual F-tests. Later, the test was extended including the LSTAR2 model [6]. Considering the similarity between LSTAR and ASTAR in powers of the threshold variable, one can assume the possible choice of ASTAR when Terasvirta test indicates LSTAR, but different switching rates can be economically explained.

To illustrate the usefulness of the STAR model with the developed asymmetric transition function, the publicly available data on the employed in Latvia in the age from 15 to 74 (in thousands) published by the Central Statistical Bureau are used [11]. The data are taken for the period from January 2002 to January 2025, and they are seasonally adjusted. The growth of employment (or decrease, if negative) is considered in this study.

Inspecting the shape of the growth data in Fig.3, we notice that the series has a gradual decline and returned during the years of large recession. It looks like nonstationary, with at least one break, may be several. Such heterogeneous dynamics may require a threshold model with a smooth transition.

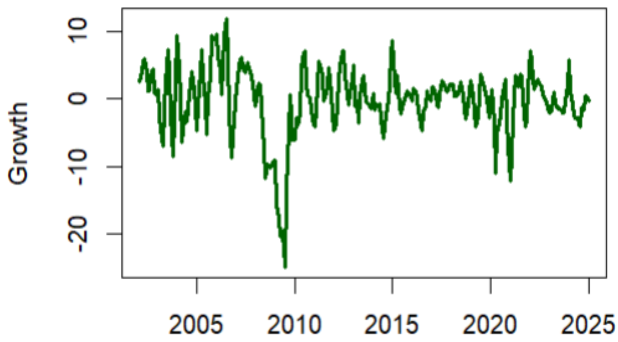

Fig. 3. **Employment growth in Latvia (in thousands) (2002M2-2025M1)**

Results and discussion

The data are found to be stationary with a break according to the Zivot-Andrews test with a potential break in April 2008. Analysis of correlations indicates dependence on the four previous values. Out of linear models, the seasonal autoregressive moving average SARIMA(3,0,4)(0,0,1)$_{12}$ model with zero mean is the best, despite it being rather long. The linear autoregressive model with four lags AR(4) with zero mean is found to be the best nonseasonal ARIMA model.

According to the Terasvirta (1994) test, linearity is rejected (with p-value = 0.00025) and the LSTAR type model is chosen as the recommended STAR model. However, the best model of each type (LSTAR, LSTAR2, ESTAR, ASTAR, SETAR) is found for the employment growth for the comparative analysis.

The evaluation of the chosen STAR model and its implementation in the used software technically is not straightforward. In the publicly available R software, the popular package *tsDyn* [12] for nonlinear time series models with regime switching provides the function *lstar*() for evaluation of the LSTAR model only with no other transition functions. If a different transition function should be used, a new estimation function should be created. Fortunately, R is software with an open code, so the *lstar*() function can be used as a basis for new functions. In addition to the LSTAR2 estimation procedure previously developed in this way, the new function for estimation of ESTAR is built for this study, and then the function for the asymmetric ASTAR model as well. Various modifications of the *lstar*() function were made for this purpose. LSTAR is of sigmoid type, but the rest functions are of ‘v’ type, which demanded the changes. Also, the number of thresholds and the number of smoothness parameters needed correction, which also influenced the creation of the Jacobian matrix used for optimisation of the parameters of the model. Then, the related prediction function required adjustment.

The first lag of the employed growth is used as the threshold variable in all estimated models. Maximum four lags are used in regime equations, which appeared the best choice. The STAR models are found to be better if only two lags are taken in one of the regimes.

Table 1 shows the comparison of the built models for the employment growth with Akaike Information Criterion (AIC), Root Mean Squared Error (RMSE), Mean Absolute Percentage Error (MAPE) for their in-sample forecast, and the residual variance. ASTAR seems to be the best having the least RMSE (2.36292), MAPE (117.5%) and the residual variance (5.502), LSTAR is the best of the rest having the least AIC (488), the second largest RMSE (2.37068), MAPE (118.1%) and the residual variance (5.539), and all STAR models are superior to the linear autoregression having the largest AIC(1284.9), RMSE (2.43912) and the residual variance (5.894). Only AIC of ASTAR (489) is a bit worse than for LSTAR (488), which can be explained by the fact that ASTAR has one more estimated parameter (two smoothing parameters in place of one). All the compared models are tested to be valid, not having unit roots.

Also, the other characteristics of the models are tested including the significance of the estimated coefficients. AR(4) has all the parameters being highly significant, but STAR models – most of them. Unfortunately, the significance of the smoothness parameters in ASTAR, LSTAR and LSTAR2 is not high, while the threshold estimates are very significant. Only the ESTAR model has all its coefficients significant, including γ and the threshold. Moreover, γ is not large (0.83), which implies a slow transition. ASTAR estimates of γ_1, γ_2 are 0.11 and 11.26 respectively, which are rather different, so the pace of transitions in different directions differ. The estimated threshold of ASTAR is -4.76, which means that if a decrease in employment is 4.76, the model is the most different.

Table 1

Comparison of models with AIC, RMSE, MAPE, and the residual variance

Model	AIC	RMSE	MAPE	Residual Variance
AR(4)	1284.9	2.43912	-	5.894
SETAR(4,4)	497	2.37485	124.3%	5.803
LSTAR(2,4)	**488**	2.37068	118.1%	5.539
LSTAR2(4,2)	500	2.41127	123.2%	5.73
ESTAR(4,2)	497	2.40734	122.1%	5.711
ASTAR(4,2)	489	**2.36292**	**117.5%**	**5.502**

Table 2 shows the comparison of out-of-sample forecast RMSE for different horizons. Root Mean Squared Error of forecast with the ASTAR model appears the least on average (1.994), despite it is not the best for most of the horizons, pointing to the best prediction ability on average among the tested models for employment growth. This suggests that the forecast made with ASTAR can be more realistic.

Table 2

Comparison of RMSE of out-of-sample forecasts of the models for different forecast horizons

Model/Forecast Horizon	4	6	9	12	Average
AR(4)	**0.96744**	1.24257	**2.69365**	3.62306	2.13168
SETAR(4,4)	1.28203	**0.78191**	2.93934	3.04262	2.01147
LSTAR(2,4)	0.98241	0.8505	2.78324	3.41134	2.00687
LSTAR2(4,2)	1.38436	0.82455	2.75560	3.81761	2.19553
ESTAR(4,2)	1.07926	1.19654	2.69783	3.60155	2.1438
ASTAR(4,2)	1.32327	0.88371	2.80977	**2.95943**	**1.99405**

Conclusions

1. This study introduces the Asymmetric Smooth Transition Autoregressive (ASTAR) model, which improves traditional STAR models by introducing an asymmetric transition function. Unlike popular LSTAR and ESTAR models, ASTAR permits different transition speeds in opposite

directions, making modelling economic data with structural shifts more flexible. The ASTAR model estimation tool has been obtained by modifying the LSTAR estimation procedure in R, which allows for a direct comparison with existing STAR models.

2. Empirical analysis of Latvian employment growth data demonstrates that ASTAR provides a better fit compared to traditional STAR models, which appears in lower RMSE (2.36292), MAPE (117.5%), and residual variance (5.502) values, as it is shown in Table 1. Although the ASTAR model has an additional smoothness parameter, resulting in a slightly higher AIC (488), its ability to model asymmetric regime shifts results in improved forecasting performance. These results underline the importance of accounting for asymmetric transitions in economic modelling. By allowing for different transition speeds, ASTAR offers a more precise representation of economic fluctuations, improving theoretical understanding and practical forecasting applications.
3. Future research could explore its application to other economic and financial indicators and improve estimation techniques. The limitations for parameters could be implemented more effectively. The characteristics of the newly introduced asymmetric transition function require a more thorough analysis with possible adjustments if needed.

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